Polarization Correlations in Atomic Photoeffect*

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The seven possible polarization correlations between incident photon and ejected electron in K-shell photoeffect have been obtained with numerical methods for the range of elements from $Z=13$ to $Z=92$ and the range of energies from 200 keV to 2 MeV. The photoeffect can serve as a polarizer of electrons, a transmitter of polarization from photons to electrons, or an analyzer of polarized radiation. In heavy elements at suitable angles of emission all these correlations can be large. Previous work on the correlations is discussed.

I. INTRODUCTION

THE atomic photoelectric effect can serve as an analyzer of polarized radiation, a transmitter of polarization from photons to electrons, or a polarizer of electrons. The preferential ejection of photoelectrons in the plane defined by the momentum **k** and polarization vector **e** of linearly polarized photons has long been known.[1] In recent years the high degree of correlation between circularly polarized photons and longitudinally polarized electrons has been discussed.[2–6] The photoeffect can also serve as a polarizer, producing transversely polarized electrons from unpolarized radiation.[5] In practice not too much use has been made of the photoeffect for these purposes, due both to experimental problems and to the lack of precise knowledge of the expected correlations.[7]

We have recently completed a numerical calculation of K-shell photoelectric cross sections, covering the range of elements from $Z=13$ to $Z=92$ and the range of energies from 200 keV to 2 MeV, neglecting electron screening. Our numerical methods and the resulting differential and total cross sections have been reported elsewhere[8]; the purpose of this paper is to present and discuss the results which we have obtained for all the possible polarization correlations between incident photon and ejected electron. For a qualitative understanding of the nature of the correlations we give a set of figures showing the energy dependence of the correlations in a high-Z element; these are found in the separate sections in which we discuss each correlation. We have appended to this paper a representative set of tables for the element $Z=84$; a complete set of tables is also available.[9]

To define these correlations we need to introduce the usual polarization parameters. We describe photon polarization with the quantities

$$\xi_1=e_1{}^*e_1-e_2{}^*e_2\,,\quad \xi_2=e_1e_2{}^*+e_2e_1{}^*\,,\quad \xi_3=i(e_1e_2{}^*-e_2e_1{}^*)\,. \tag{1}$$

Here e_1 is the component of the photon polarization vector **e** which lies in the plane of the incoming photon **k** and outgoing electron **p**, and e_2 is the component perpendicular to the plane (along $\mathbf{k}\times\mathbf{p}$), so that $(e_1,e_2,\mathbf{k})$ form a righthanded set. Photons linearly polarized parallel or perpendicular to the production plane are characterized by a nonvanishing ξ_1, circularly polarized photons by nonvanishing ξ_3. Similarly, we describe the polarization state of the ejected electron in terms of the direction $\boldsymbol{\zeta}$ of its spin in its rest system; ζ_3 is taken along the *electron* direction **p**, ζ_1 in the scattering plane, and ζ_2 perpendicular to the scattering plane (along $\mathbf{k}\times\mathbf{p}$), so that $(\zeta_1,\zeta_2,\zeta_3)$ again form a right-handed set. Longitudinally polarized electrons are characterized by nonvanishing ζ_3. It should be noted that the coordinate systems used for the description of photon and electron polarizations are different.

In terms of these parameters the differential cross section for K-shell photoeffect, summed over both K-shell electrons, is of the form

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol.}}\left[\tfrac{1}{2}\sum_{i,j=0}\xi_i\zeta_jC_{ij}\right], \tag{2}$$

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where $\xi_0=\zeta_0=C_{00}=1$, and $(d\sigma/d\Omega)_{\text{unpol.}}$ is the differential cross section from unpolarized photons, summed over final electron spins. The quantities C_{ij}, satisfying $|C_{ij}|\leq 1$, are the polarization correlations with which this paper is concerned.

It would appear from Eq. (2) that, not counting $C_{00}\equiv 1$, there are fifteen polarization correlations. But in fact invariance considerations permit only seven nonzero correlations. This follows most directly from the fact that the matrix element

$$H=-e(2\pi/k)^{1/2}\int d^3r\psi_{\text{fin}}{}^{\dagger}\boldsymbol{\alpha}\cdot\mathbf{e}e^{i\mathbf{k}\cdot\mathbf{r}}\psi_{\text{in}} \qquad (3)$$

is invariant under the substitutions

$$\psi\rightarrow\sigma_2\psi\,,\quad (e_1,e_2)\rightarrow(-e_1,\,e_2)\,. \qquad (4)$$

For we may absorb these substitutions into the polarization parameters by

$$(\xi_1,\xi_2,\xi_3)\rightarrow(\xi_1,\,-\xi_2,\,-\xi_3)\,,$$
$$(\zeta_1,\zeta_2,\zeta_3)\rightarrow(-\zeta_1,\,\zeta_2,\,-\zeta_3)\,, \qquad (5)$$

and similarly for the bound-state polarization parameters which are summed out. If Eq. (2) is to be invariant under the substitutions (5), then we conclude that, not counting $C_{00}\equiv 1$, the only nonzero correlations are

$$C_{02},\,C_{10},\,C_{12},\,C_{21},\,C_{23},\,C_{31},\,C_{33}\,. \qquad (6)$$

These seven correlations are indeed all nonzero, and they will be discussed in the subsequent sections. C_{02} produces transversely polarized electrons from unpolarized photons, C_{10} is the well-known correlation which analyzes linearly polarized photons, and C_{33} converts circularly polarized photons to longitudinally polarized electrons.

We have written down in Eqs. (2.25)–(2.27) of Ref. 8 expressions for the C_{ij} in terms of reduced partial-wave matrix elements R_κ and continuum wave-function phase shifts δ_κ; it is from these equations that the results presented in this paper were calculated. Some information which can be deduced from these equations concerning the behavior of the correlations at small angles should be mentioned. First of all, since the angular distribution $(d\sigma/d\Omega)_{\text{unpol.}}$ does not vanish for forward (and backward) angles (unlike the Sauter distribution,[1,10] valid only to lowest order in $a\equiv Z\alpha$), one sees that all correlations except C_{33} vanish in the forward and backward directions, and $C_{33}(0°)=+1$, $C_{33}(180°)=-1$. Further, for small angles the correlations C_{02}, C_{12}, C_{21}, and C_{31} go as θ, while C_{10} and C_{23} go as θ^2. And finally, for small angles one has the identity $C_{12}=-C_{21}$. However, one can see from the figures and tables that small angles, at these energies, means much less than 5°, and indeed all the correlations undergo very rapid and violent changes over this region of very small angles, in contrast with their comparatively slow variations through the main angular regions. Indeed, in the figures, we have not attempted to draw in the variation of the correlations between 0 and 5°.

Before proceeding to the specific correlations we may also discuss their behavior at low and at high energies and as a function of Z. In the nonrelativistic theory, $\boldsymbol{\alpha}\cdot\mathbf{e}$ in Eq. (3) is replaced by $\mathbf{p}\cdot\mathbf{e}$, and all dependence on electron spin disappears. This leads to a differential cross section proportional to $(1+\xi_1)$, so that $C_{10}=1$ for all angles and all other correlations vanish. Of course even at threshold there are relativistic corrections of relative order a^2 [which for example cause $C_{10}(0°)=0$], and away from threshold there are relativistic corrections of relative $O(1)$, as given by Sauter,[1,10] which in fact for the larger angles cause the preferred direction of emission to be perpendicular to rather than in the plane of $\mathbf{k}$ and $\mathbf{e}$. In the relativistic theory correlations with the electron spin can occur. In addition to C_{10} the correlations C_{31} and C_{33} are also $O(1)$, while the remaining correlations are $O(a)$ and so mainly important only in heavy elements. At very high energies one can show[6,11] that for angles of significant electron emission the only nonvanishing correlations are C_{12}, C_{21}, and C_{33}, and further $C_{33}=+1$ and $C_{12}=-C_{21}$.

II. THE CORRELATION C_{02}

The correlation C_{02} permits the production of transversely polarized electrons from unpolarized photons; the resulting electron spin is perpendicular to the production plane. The correlation vanishes both in the nonrelativistic and extreme relativistic limits and is $O(a)$. Figure 1 shows C_{02} for $Z=92$ at some representative energies. Like the more familiar correlation C_{10}, C_{02} exhibits a "crossover" feature: For energies above some minimum energy the spin direction $\mathbf{k}\times\mathbf{p}$ is favored at smaller angles and $-\mathbf{k}\times\mathbf{p}$ at larger angles. Previous work on this correlation is due to Kolbenstvedt and Olsen[12] and Nagel,[5] who obtained the analytic expression for the correlation to lowest nonvanishing order in a, and show curves which are qualitatively similar to ours. [These curves fail near the forward direction where the angular distribution itself is $O(a^2)$ rather than $O(1)$; the same comment will apply to all the analytical calculations subsequently mentioned.] Numerical results at threshold have been reported by Nagel and Olsson.[13] In the report on their numerical calculation of K-shell cross sections Hultberg, Nagel, and Olsson[14] show one representative curve for C_{02}

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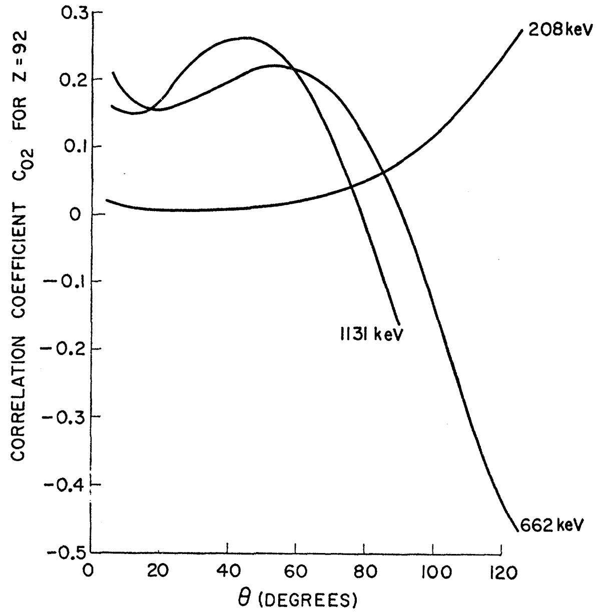

FIG. 1. Correlation C_{02} for the production of transversely polarized electrons from unpolarized photons, as a function of angle for $Z=92$ at several energies.

(P_1 in their notation), and in unpublished work[15] they have given curves for $Z=82$ and $Z=92$ over a range of energies up to 662 keV, which are in good agreement with the corresponding results of our tables. As has been noted,[5,12] the electron polarization which C_{02} implies is analogous to that produced in relativistic Coulomb scattering with unpolarized electrons, and can be analyzed in the same way; this correlation has the advantage[12] of not vanishing as rapidly at high energies.

III. THE CORRELATION C_{10}

The correlation C_{10}, as already noted, reflects the fact that the photoeffect can serve as an analyzer for photons linearly polarized parallel or perpendicular to the production plane. In the nonrelativistic theory[1] $C_{10}=+1$, which says no electrons are emitted perpendicular to the polarization direction of the photons. Of course, even at threshold there are relativistic corrections of $O(a^2)$, which, for example, cause $C_{10}(0°)=0$. The calculation of C_{10} to lowest order in a in the relativistic theory was made by Sauter,[1,10] and like C_{02} it exhibits a crossover feature: Above a minimum energy, emission in the plane of $\mathbf{k}$ and $\mathbf{e}$ (as predicted by the nonrelativistic theory) is favored for smaller angles of emission, whereas emission *perpendicular* to the plane is favored at larger angles. As discussed by Olsen,[2] in lowest order in a the minimum electron energy for the crossover feature is $\epsilon=\frac{5}{3}$ (in units $\hbar=c=m_e=1$), i.e., ≈ 340 keV for the photon, and at higher energies the crossover angle θ_c is determined from

$$k(k+1)(1-\beta\cos\theta_c)=2, \qquad (7)$$

where k is the photon energy in the same units. Olsen also noted that in the same approximation, at the crossover angle θ_c such that $C_{10}(\theta_c)=0$, electrons from *circularly* polarized photons are *completely* polarized along the *photon* direction. We shall return to this feature in our discussion of C_{31} and C_{33}. We should also note that the analytical corrections to C_{10} of order a were obtained by Gavrila[16] and Nagel.[5]

Results of our numerical calculation of C_{10} are given in the tables appended to this paper and in Fig. 2. We also give in Table I some information on the crossover angle θ_c as a function of k and Z. At low energies, the correlation is quite close to its nonrelativistic value except at large angles (and extremely close to the forward direction). As the energy increases the correlation appears to decrease monotonically for the angles of significant electron emission, and it will vanish in the high-energy limit. However, C_{10} can still be quite sizeable at 2 MeV. Table I shows that in heavy elements the crossover angle θ_c begins to vary significantly from the lowest order prediction Eq. (7); the crossover occurs significantly *later*.

Among the seven photoeffect polarization correlations, only C_{10} appears to have so far been the subject

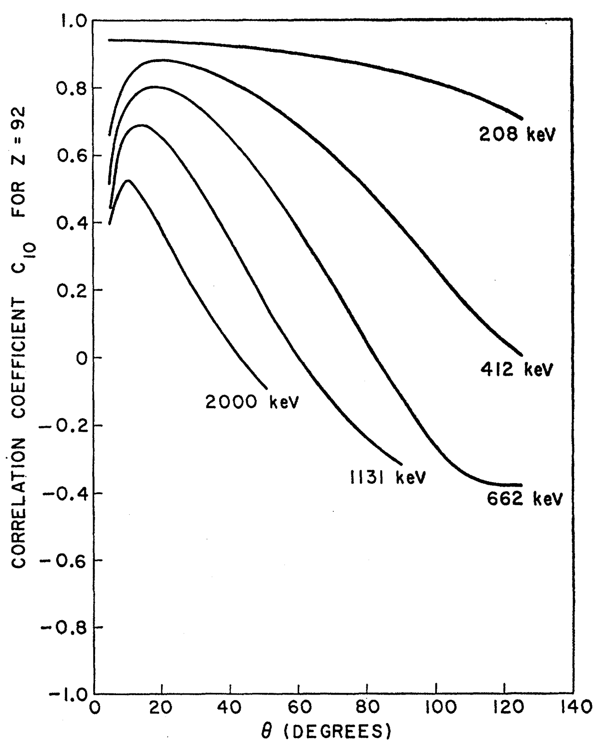

FIG. 2. Correlation C_{10} for the analysis of linearly polarized photons, as a function of angle for $Z=92$ at several energies.

[15] S. Hultberg, B. Nagel, and P. Olsson (unpublished). (Dr. Nagel now informs me that they have made calculations for all seven polarization correlations.)

[16] M. Gavrila, Phys. Rev. 113, 514 (1959).

of experimental investigations. In the energy range of interest there are experiments by Hereford and co-workers[17] and by Brini *et al.*[18] In both cases the objective was to test the predicted crossover feature of Sauter's relativistic theory; the first group appeared to see such a feature while the second group did not. Now that a more precise theory is available, and especially in view of the shift of the predicted θ_c as shown in Table I, further experiments on this question would seem desirable.

IV. THE CORRELATION C_{12}

The correlation C_{12} alters the production of transversely polarized electrons from that given by C_{02} in the case that the initial photons are linearly polarized parallel or perpendicular to the production plane. To determine C_{12} will require also determining C_{02} and C_{10}, or at least designing a set of experiments which subtract out their effect; C_{12} is the only one of the seven correlations which cannot in this sense be determined independently. Like C_{02}, the correlation C_{12} vanishes in the nonrelativistic limit and is $O(a)$; unlike C_{02} it remains finite in the extreme relativistic limit. Connections between C_{12} and C_{21} have already been mentioned: $C_{12}=-C_{21}$ for very small angles and also for all angles of significant electron emission (which are also small angles, but not as small) in the extreme relativistic limit.[6,11] There does not appear to be any discussion of the general analytic form of C_{12}, although an expression to lowest nonvanishing order in a could be extracted from Nagel's work.[5] An expression for the lowest order result valid in the extreme relativistic limit was obtained by Banerjee.[3] The complete analytical expression for the relativistic limit has now been given by Nagel,[11] and he has evaluated it numerically for three choices of Z. Results of our calculation of C_{12} are detailed in the Appendix and in Fig. 3. Since the correlation is $O(a)$ it is most important in heavy elements, but it can become quite sizeable for intermediate Z at back angles (and Nagel's work[11] indicates that in the extreme relativistic case it is greater for intermediate than for large Z). C_{12} is always positive in the regions considered here, but Nagel's work[11] indicates that for very heavy Z and high energies there will be a crossover. Finally, for heavy Z and intermediate energies C_{12} is large, especially at the more backward angles, and so produces major effects in the production of transversely polarized electrons from linearly polarized photons.

TABLE I. Crossover angle θ_c for which $C_{10}(\theta_c)=0$ as a function of charge Z and photon energy k(MeV).

k \ Z	13	26	50	84
0.354	≈134°	≈132°	≈133°	≈157°
0.662	≈74°	≈68°	70°	80°
1.131	42°	41°	44°	56°
1.5	≈35°	≈35°	36°	47°
2.0	≈25°	≈24°	31°	40°

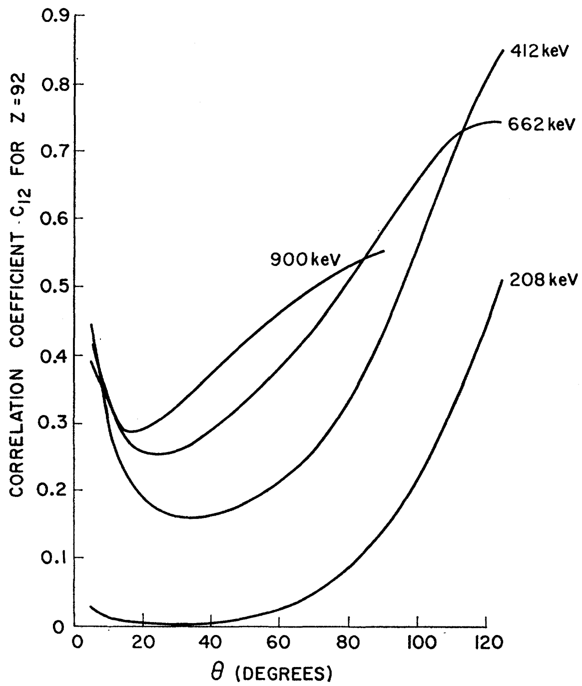

FIG. 3. Correlation C_{12} for the production of transversely polarized electrons from linearly polarized photons, as a function of angle for $Z=92$ at several energies.

V. THE CORRELATION C_{21}

The correlation C_{21}, like C_{12}, produces transversely polarized electrons from linearly polarized photons. However this correlation, also $O(a)$, is between electrons polarized *in* the production plane and photons polarized at 45° to the production plane. As we have said before, $C_{12}=-C_{21}$ for very small angles and also in the extreme relativistic limit. The remarks on previous work also apply to both processes: from Nagel's work[5] one can extract an analytic expression for the lowest nonvanishing order in a and from his more recent work[11] one of course has the extreme relativistic limit ($C_{21}=-C_{12}$, note there is a difference in sign and in his choice of coordinate system). Results for C_{21} are in the Appendix and in Fig. 4. Remarks on magnitudes apply to both correlations and need not be repeated here. However, the correlation C_{21} exhibits, even at the lowest energies, a crossover feature; the crossover angle increases with energy, unlike C_{02} or C_{10}. Since C_{21} is large at intermediate energies for back angles the photoeffect provides a fairly efficient mechanism for the transfer of these polarizations.

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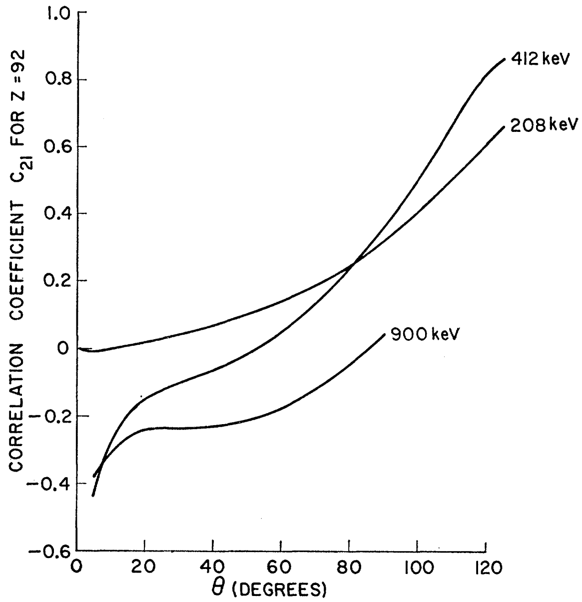

FIG. 4. Correlation C_{21} for the production of transversely polarized electrons from linearly polarized photons, as a function of angle for $Z=92$ at several energies.

VI. THE CORRELATION C_{23}

The correlation C_{23}, which is $O(a)$, produces longitudinally polarized electrons from linearly polarized photons. The photon polarization should again be at 45° to the production plane. There does not appear to have ever been any analytical discussion of this correlation, although the result to lowest order in a can again be extracted from Nagel's work.[5] Results for C_{23} are given in the Appendix and in Fig. 5. In the range considered here the correlation is always negative. And although C_{23} is $O(a)$ and vanishes in the nonrelativistic and extreme relativistic limits, for intermediate and large Z at intermediate energies it becomes quite sizeable at large angles and so provides a fairly efficient mechanism for the transfer of these polarizations.

VII. THE CORRELATION C_{31}

The correlation C_{31} produces electrons transversely polarized in the production plane from circularly polarized photons. The correlation vanishes in both the nonrelativistic and extreme relativistic limits, but it is $O(1)$ and very large for intermediate energies and angles. Figure 6 shows C_{31} for $Z=92$ at some representative energies; further results are given in the Appendix. At very forward angles the correlation is positive, but it soon crosses over and remains negative through most of the angular range, with a broad minimum at intermediate angles. (The curves suggest that there could actually be an energy, near 600 keV, and angle, near 80°, for which $C_{31}=-1$.) Analytical expressions for C_{31} to lowest order in a were obtained by Olsen[2] and Banerjee[3]; Nagel[5] also obtained terms of relative order a. Numerical results obtained by Hultberg, Nagel, and Olsson[14,15] (in their notation $C_{31}=-P_3$) are in good accord with the corresponding values of our tables. Numerical results at threshold were given by Nagel and Olsson.[13]

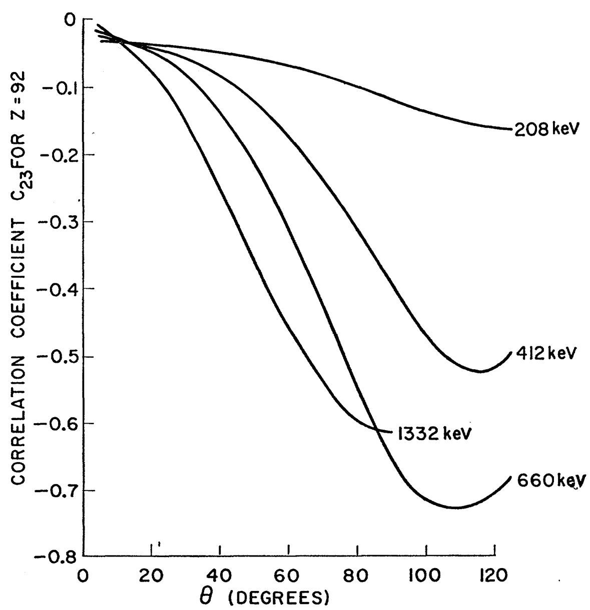

FIG. 5. Correlation C_{23} for the production of longitudinally polarized electrons from linearly polarized photons, as a function of angle for $Z=92$ at several energies.

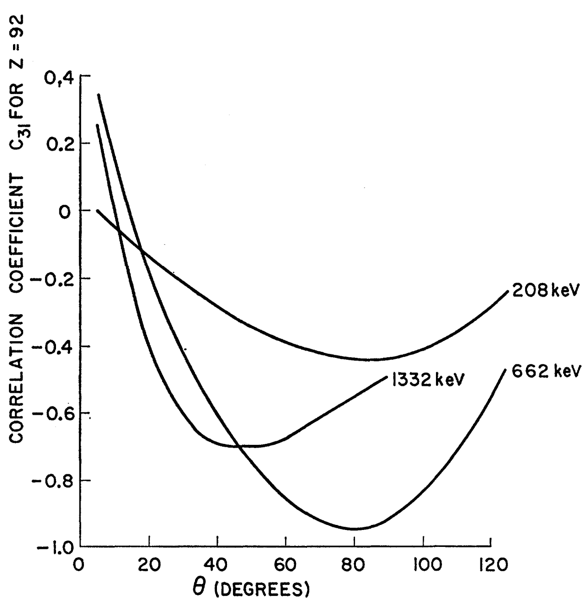

FIG. 6. Correlation C_{31} for the production of transversely polarized electrons from circularly polarized photons, as a function of angle for $Z=92$ at several energies.

TABLE II. $D(\theta_c)$ as defined by Eq. (8) for $Z=84$ as a function of photon energy k (in MeV).

k	0.662	1.131	1.5	2.0
$D(\theta_c)$	0.963	0.972	0.965	0.961

It was noted by Olsen[2,4] that, to lowest order in a, at the crossover angle θ_c for which $C_{10}(\theta_c)=0$, electrons from circularly polarized photons are completely polarized along the *photon* direction, i.e.,

$$D(\theta_c)\equiv C_{33}(\theta_c)\cos\theta_c - C_{31}(\theta_c)\sin\theta_c = 1. \quad (8)$$

For this to be true requires that, in the language of Eqs. (2.25)–(2.27) of Ref. 8,

$$|K_-|^2=|J_+|^2=0 \quad \text{when} \quad \mathrm{Re}[J_-{}^*J_+ + K_-{}^*K_+]=0; \quad (9)$$

it does not seem that this can in general be true. We show, however, in Table II the calculation of $D(\theta_c)$ for $Z=84$ from our results; the deviations from Olsen's prediction are very small, and almost within the errors of the calculation.

VIII. THE CORRELATION C_{33}

The correlation C_{33} is a transmitter of helicity, producing longitudinally polarized electrons from circularly polarized photons. The only one of the seven correlations which does not vanish in forward and backward directions $[C_{33}(0°)=-C_{33}(180°)=1]$, it vanishes in the nonrelativistic limit, but becomes 1 for all angles of significant electron emission in the extreme relativistic limit.[6] We show results for C_{33} in the tables appended to the paper and in Fig. 7. Even at intermediate energies the correlation is large; it appears to approach one at high energies somewhat more rapidly in the lighter elements. Since C_{33} ranges from $+1$ to -1 it exhibits a crossover, which, however, at high energies moves to very backward angles. Analytical expressions for C_{33} to lowest order in a were obtained by Olsen,[2] by Banerjee,[3] and by Fano, McVoy, and Albers[4]; Nagel[5] also obtained terms of relative order a. The extreme relativistic limit of C_{33} was discussed by Pratt.[6] Like C_{02} and C_{31}, numerical results obtained for C_{33} by Hultberg, Nagel, and Olsson[14,15] are in good accord with the corresponding values of our tables. Nagel and Olsson[13] have given numerical results at threshold. The connection between the crossover angle for C_{10} and production of completely polarized electrons from circularly polarized photons has been discussed in the previous section.

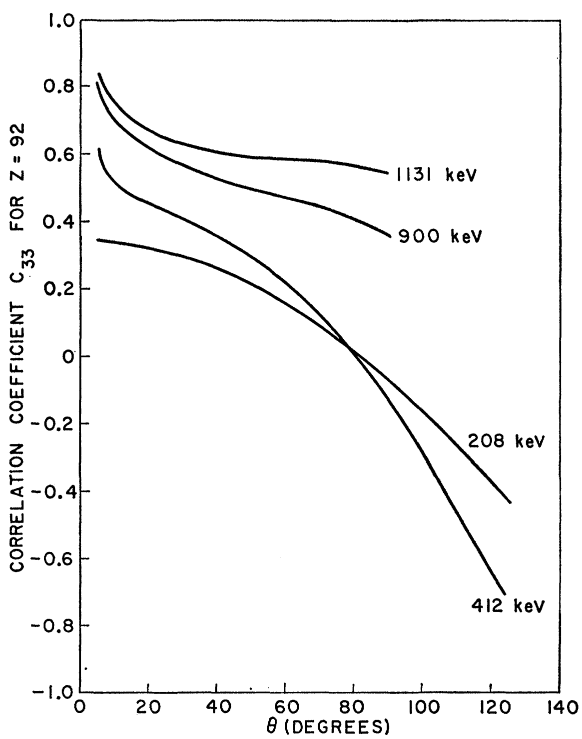

FIG. 7. Correlation C_{33} for the production of longitudinally polarized electrons from circularly polarized photons, as a function of angle for $Z=92$ at several energies.

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APPENDIX: TABLES III–VII

TABLE III. Polarization correlations for $Z=84$ at 354 keV as a function of angle.

θ (deg)	C_{02}	C_{10}	C_{12}	C_{21}	C_{23}	C_{31}	C_{33}
0	0.000	0.000	0.000	0.000	0.000	0.000	1.000
5	0.26	0.77	0.4	−0.4	−0.0	0.3	0.5
10	0.16	0.88	0.24	−0.2	−0.0	0.0	0.4
15	0.12	0.90	0.18	−0.16	−0.0	−0.0	0.4
20	0.10	0.90	0.15	−0.12	−0.0	−0.15	0.4
25	0.10	0.90	0.13	−0.09	−0.04	−0.22	0.38
30	0.09	0.88	0.12	−0.07	−0.05	−0.29	0.36
35	0.10	0.87	0.12	−0.05	−0.06	−0.36	0.34
40	0.10	0.85	0.12	−0.03	−0.07	−0.42	0.31
45	0.10	0.83	0.13	−0.01	−0.08	−0.48	0.28
50	0.10	0.81	0.13	0.01	−0.10	−0.53	0.25
55	0.11	0.78	0.14	0.03	−0.12	−0.58	0.21
60	0.11	0.75	0.16	0.06	−0.14	−0.63	0.18
65	0.11	0.72	0.18	0.09	−0.16	−0.67	0.13
70	0.12	0.68	0.20	0.13	−0.18	−0.71	0.08
75	0.12	0.65	0.22	0.18	−0.21	−0.74	0.03
80	0.12	0.61	0.26	0.22	−0.24	−0.76	−0.02
85	0.12	0.56	0.30	0.28	−0.27	−0.78	−0.08
90	0.12	0.51	0.35	0.34	−0.30	−0.78	−0.15
95	0.12	0.46	0.40	0.41	−0.33	−0.78	−0.22
100	0.12	0.41	0.46	0.48	−0.35	−0.76	−0.30
105	0.12	0.36	0.53	0.56	−0.38	−0.73	−0.38
110	0.11	0.31	0.60	0.64	−0.40	−0.69	−0.46
115	0.11	0.26	0.68	0.72	−0.41	−0.63	−0.54
120	0.10	0.21	0.75	0.79	−0.41	−0.56	−0.62
125	0.10	0.16	0.81	0.86	−0.41	−0.49	−0.70

TABLE IV. Polarization correlations for $Z=84$ at 662 keV as a function of angle.

θ	C_{02}	C_{10}	C_{12}	C_{21}	C_{23}	C_{31}	C_{33}
0	0.000	0.000	0.000	0.000	0.000	0.000	1.000
5	0.218	0.57	0.40	−0.38	−0.04	0.294	0.74
10	0.177	0.78	0.30	−0.27	−0.04	0.058	0.62
15	0.161	0.81	0.264	−0.231	−0.03	−0.102	0.56
20	0.151	0.80	0.252	−0.221	−0.05	−0.222	0.54
25	0.144	0.77	0.241	−0.198	−0.07	−0.34	0.53
30	0.159	0.73	0.236	−0.157	−0.09	−0.46	0.50
35	0.190	0.69	0.250	−0.14	−0.10	−0.56	0.46
40	0.195	0.63	0.276	−0.16	−0.13	−0.64	0.44
45	0.174	0.56	0.292	−0.16	−0.18	−0.71	0.42
50	0.184	0.49	0.298	−0.1	−0.22	−0.78	0.38
55	0.225	0.42	0.316	−0.1	−0.24	−0.82	0.36
60	0.21	0.336	0.352	−0.1	−0.30	−0.87	0.34
65	0.14	0.233	0.386	−0.1	−0.36	−0.92	0.30
70	0.13	0.160	0.410	−0.1	−0.41	−0.94	0.26
75	0.16	0.109	0.42	0.1	−0.45	−0.93	0.24
80	0.12	0.001	0.46	0.1	−0.51	−0.94	0.22
85	−0.01	−0.140	0.52	0.1	−0.56	−0.95	;.15
90	−0.06	−0.196	0.54	0.2	−0.60	−0.92	0.08
95	−0.05	−0.18	0.55	0.3	−0.65	−0.86	0.07
100	−0.12	−0.28	0.59	0.3	−0.69	−0.82	0.04
105	−0.27	−0.4	0.66	0.4	−0.7	−0.76	−0.08
110	−0.34	−0.4	0.69	0.5	−0.7	−0.69	−0.19
115	−0.4	−0.4	0.68	0.5	−0.7	−0.63	−0.2
120	−0.4	−0.4	0.68	0.5	−0.7	−0.54	−0.25
125	−0.5	−0.4	0.7	0.6	−0.6	−0.43	−0.4
(deg)							

TABLE V. Polarization correlations for $Z=84$ at 1131 keV as a function of angle.

θ (deg)	C_{02}	C_{10}	C_{12}	C_{21}	C_{23}	C_{31}	C_{33}
0	0.000	0.000	0.000	0.000	0.000	0.000	1.000
5	0.18	0.48	0.42	−0.41	−0.019	0.263	0.82
10	0.15	0.67	0.34	−0.33	−0.035	0.007	0.79
15	0.15	0.68	0.30	−0.27	−0.051	−0.210	0.70
20	0.17	0.63	0.30	−0.25	−0.072	−0.379	0.67
25	0.19	0.56	0.32	−0.26	−0.098	−0.507	0.65
30	0.21	0.48	0.35	−0.27	−0.136	−0.602	0.63
35	0.23	0.38	0.37	−0.27	−0.18	−0.675	0.62
40	0.23	0.28	0.38	−0.24	−0.23	−0.73	0.61
45	0.22	0.19	0.38	−0.22	−0.27	−0.75	0.61
50	0.22	0.10	0.40	−0.22	−0.32	−0.76	0.61
55	0.20	0.01	0.4	−0.22	−0.39	−0.76	0.61
60	0.16	−0.06	0.4	−0.18	−0.43	−0.75	0.60
65	0.11	−0.13	0.4	−0.13	−0.45	−0.74	0.61
70	0.07	−0.19	0.4	−0.1	−0.49	−0.70	0.61
75	0.01	−0.24	0.4	−0.1	−0.55	−0.66	0.61
80	−0.06	−0.28	0.4	−0.1	−0.58	−0.64	0.59
85	−0.13	−0.32	0.4	0.0	−0.58	−0.62	0.57
90	−0.19	−0.36	0.4		−0.59	−0.57	0.56

TABLE VI. Polarization correlations for $Z=84$ at 1500 keV as a function of angle.

θ (deg)	C_{02}	C_{10}	C_{12}	C_{21}	C_{23}	C_{31}	C_{33}
0	0.000	0.000	0.000	0.000	0.000	0.000	1.000
5	0.15	0.45	0.43	−0.43	−0.01	0.22	0.85
10	0.14	0.61	0.35	−0.34	−0.03	−0.05	0.79
15	0.14	0.58	0.32	−0.29	−0.06	−0.29	0.76
20	0.18	0.49	0.32	−0.26	−0.09	−0.46	0.73
25	0.21	0.40	0.35	−0.28	−0.12	−0.56	0.72
30	0.22	0.30	0.40	−0.33	−0.16	−0.61	0.72
35	0.21	0.20	0.42	−0.32	−0.23	−0.65	0.72
40	0.22	0.11	0.38	−0.24	−0.28	−0.67	0.71
45	0.23	0.03	0.4	−0.2	−0.29	−0.64	0.73
50	0.19	−0.05	0.4		−0.33	−0.62	0.74
55	0.1	−0.1	0.4		−0.41	−0.62	0.73
60	0.1	−0.1	0.4		−0.4	−0.59	0.73
65		−0.2	0.3		−0.4	−0.53	0.76
70		−0.2	0.3		−0.4	−0.49	0.76

TABLE VII. Polarization correlations for $Z=84$ at 2000 keV as a function of angle.

θ (deg)	C_{02}	C_{10}	C_{12}	C_{21}	C_{23}	C_{31}	C_{33}
0	0.000	0.000	0.000	0.000	0.000	0.000	1.000
5	0.14	0.42	0.46	−0.46	−0.01	0.17	0.88
10	0.12	0.53	0.37	−0.36	−0.04	−0.11	0.83
15	0.13	0.45	0.32	−0.29	−0.08	−0.36	0.82
20	0.18	0.32	0.29	−0.23	−0.11	−0.51	0.80
25	0.22	0.23	0.35	−0.3	−0.13	−0.54	0.80
30	0.20	0.14	0.46	−0.4	−0.19	−0.55	0.80
35	0.2	0.05	0.4		−0.27	−0.57	0.80
40	0.2	−0.00	0.3		−0.27	−0.53	0.81
45	0.2	−0.07	0.3		−0.27	−0.47	0.84
50	0.1	−0.1			−0.3	−0.47	0.83
55	0.1	−0.1			−0.4	−0.5	0.8
60	0.1	−0.1			−0.4	−0.4	0.8
65		−0.2			−0.4	−0.5	0.8
70		−0.2			−0.4		0.8